Math 2263

Quiz 9

Name Section Score

(7 points) Identify the surface whose equation is given as $2r^2 + z^2 = 1$.

Solution: Since $2r^2 + z^2 = 1$ and $r^2 = x^2 + y^2$, we have $2(x^2 + y^2) + z^2 = 1$, an ellipsoid centered at the origin with intercepts $x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{2}}$ and $z = \pm 1$.

(8 points) Evaluate

$$\iiint_B (x^2 + y^2 + z^2)\, dV,$$

where B is the ball with center the origin and radius 5.

Solution: In spherical coordinates, B is represented by $\{(\rho, \theta, \phi) | 0 \le \rho \le 5, 0 \le \theta \le 2\pi, 0 \le \phi \le \pi\}$. Thus

$$\begin{aligned}
\iiint_B (x^2 + y^2 + z^2) dV &= \int_0^{\pi} \int_0^{2\pi} \int_0^{5} (\rho^2)\rho^2 \sin\phi \, d\rho \, d\theta \, d\phi \\
&= \int_0^{\pi} \sin\phi \, d\phi \times \int_0^{2\pi} d\theta \times \int_0^{5} \rho^4 d\rho \\
&= [-\cos\phi]_0^{\pi} \times \; [\theta]_0^{2\pi} \; \times [\frac{1}{5}\rho^5]_0^5 \\
&= (2)(2\pi)(625) \\
&= 2500\pi
\end{aligned}$$